Topcolor-Assisted Supersymmetry

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Abstract

It has been known that the supersymmetric flavor changing neutral current problem can be avoided if the squarks take the following mass pattern, namely the first two generations with the same chirality are degenerate with masses around the weak scale, while the third generation is very heavy. We realize this scenario through the supersymmetric extension of a topcolor model with gauge mediated supersymmetry breaking.

PACS numbers: 12.60.Cn, 12.60.Jv

I. INTRODUCTION

Supersymmetry (SUSY) [1] provides a solution to the gauge hierarchy problem if it breaks dynamically [2]. However, the general supersymmetric extension of the Standard Model (SM) suffers from the flavor changing neutral current (FCNC) problem [3]. The SUSY FCNC should be suppressed by certain specific mass patterns of the sleptons and squarks. The sfermion mass pattern depends on the underlying physics of SUSY breaking. For example, one of the popular choice for the sfermion mass matrix is the universality in the flavor space. Such a mass matrix can be resulted from the gauge mediated SUSY breaking (GMSB) scenario [4,5]. Here gauge means the SM gauge interactions. Another inspiring choice is that the first two generation sfermions are very heavy around $(10-100)$ TeV where the third generation sfermions are at the weak scale [6]. This kind of model is often referred to as effective SUSY. It can be realized in the GMSB scenario [7], or in that where an anomalous U(1) mediates SUSY breaking [8]. One point in this case is that the first two generations and the third generation are treated differently. For example, they maybe in different representations of some new gauge interactions mediating SUSY breaking.

In this work, we consider a sfermion mass pattern which looks opposite to that of the effective SUSY. It is that the first two generations with the same chirality are degenerate with masses around the weak scale, and the third generation is super heavy. The SUSY FCNC is also suppressed in this case [3].[1] In fact, this pattern is not new. It could be understood by an U(2) symmetry between the first two generations in the supergravity scenario [9]. In this paper an alternative origin of it will be discussed. We note that the above mass pattern can be also a result of a supersymmetric topcolor model with GMSB. Here gauge (G) means the gauge interactions of the topcolor model.

Topcolor models [10] were proposed for a dynamical understanding of the third generation

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[1] There are other viable alternatives. For example, only the squarks satisfy this mass pattern, while the slepton mass matrices follow universality.

quark masses. The basic idea is that the third generation (at least the top quark) undergoes a super strong interaction which results in a top quark condensation. The condensation gives top quark a large mass, and the bottom quark mainly gets its mass due to the instanton effect of the topcolor interactions. This top quark condensation contributes only a small part of the electroweak symmetry breaking (EWSB). The Higgs mechanism may be introduced for the EWSB. Therefore, the idea of the topcolor can be generalized into SUSY models naturally. In the scenario of the GMSB, suppose the strong topcolor gauge interaction involves the full third generation, and the first two generations participate a weaker gauge interaction universally, the above described sfermion mass pattern will be generated.

Note that the decoupling of the third generation scalars is a consistent choice in the supersymmetric topcolor models. Because the third generation quarks obtain dynamical masses, the Yukawa couplings are always small.

The whole physical picture is like as follows. At the energy scale about 10^6 GeV, SUSY breaking occurs in a secluded sector. It is mediated to the observable sector through the gauge interactions. The scale of the messengers is around 10^7 GeV. The topcolor scale is around (1 – 10) TeV. Below this scale, the gauge symmetries break into that of the SM. The resultant sparticle spectrum of the observable sector is the following. Besides the squarks, the gauginos of the super strong interaction are around 100 TeV. The gauginos of the weaker gauge interaction are at about the weak scale. The Higgs bosons for the topcolor symmetry breaking are as heavy as 100 TeV, and the Higgs bosons for the EWSB at the weak scale. By integrating out the heavy fields above 1 TeV or so, the effective theory is the ordinary (two-Higgs-doublets) topcolor model with the weak scale gauginos, Higgsinos and the first two generation squarks with degeneracy.

This paper is organized as follows. After a brief review of the topcolor model in the next section. The supersymmetric extension of the topcolor model within the framework of the GMSB is described in Sec. III. Summary and discussions are presented in Sec. IV.

II. BRIEF REVIEW OF THE TOPCOLOR MODEL

In this paper, we consider the topcolor model which, at the scale about $(1 - 10)$ TeV, has interactions [10] $SU(3)_1 \otimes SU(3)_2 \otimes U(1)_{Y_1} \otimes U(1)_{Y_2} \otimes SU(2)_L$. The fermions are assigned $(SU(3)_1, SU(3)_2, U(1)_{Y_1}, U(1)_{Y_2})$ quantum numbers as follows,

$$
\begin{aligned}
(t,b)_L &\sim (3,1,\tfrac{1}{3},0), \quad (t,b)_R \sim (3,1,(\tfrac{4}{3},-\tfrac{2}{3}),0), \\
(\nu_\tau,\tau)_L &\sim (1,1,-1,0), \quad \tau_R \sim (1,1,-2,0), \\
(u,d)_L, (c,s)_L &\sim (1,3,0,\tfrac{1}{3}), \quad (u,d)_R, (c,s)_R \sim (1,3,0,(\tfrac{4}{3},-\tfrac{2}{3})), \\
(\nu_l,l)_L\ (l=e,\mu) &\sim (1,1,0,-1), \quad l_R \sim (1,1,0,-2). \qquad (1)
\end{aligned}
$$

The topcolor symmetry breaks spontaneously to $SU(3)_1 \otimes SU(3)_2 \to SU(3)_{QCD}$ and $U(1)_{Y_1} \otimes U(1)_{Y_2} \to U(1)_Y$ through an scalar field $\Phi \sim (3,\bar{3},\frac{1}{3},-\frac{1}{3})$ which develops a vacuum expectation value (VEV). The $SU(3)_1 \otimes U(1)_{Y_1}$ are assumed to be strong which make the formation of a top quark condensate but disallow the bottom quark condensate. The bottom quark mainly gets its mass due to the $SU(3)_1$ instanton effect. The τ lepton does not condensate.

III. SUPERSYMMETRIC TOPCOLOR MODEL

In the supersymmetric extension, the gauge symmetries of the above topcolor model keep unchanged. The particle contents are given below. In addition to the superpartners of the particles described in the last section, some elementary Higgs superfields are introduced. The breaking of the topcolor symmetry needs one pair of the Higgs superfields Φ_1 and Φ_2. And the EWSB requires another pair of the Higgs superfields H_u and H_d, like in the ordinary supersymmetric SM. Their quantum numbers under the $SU(3)_1 \otimes SU(3)_2 \otimes U(1)_{Y_1} \otimes U(1)_{Y_2} \otimes SU(2)_L$ are

$$
\begin{aligned}
&\Phi_1(3,\bar{3},\tfrac{1}{3},-\tfrac{1}{3}), \quad \Phi_2(\bar{3},3,-\tfrac{1}{3},\tfrac{1}{3}), \\
&H_u(1,1,0,1,2), \quad H_d(1,1,0,-1,2). \qquad (2)
\end{aligned}
$$

The messenger sector is introduced as

$$S_1 ; S_1^0 = (1;1;1;0;2) ; \quad S_1 ; S_1^0 = (1;1; \ 1;0;2) ;$$
$$T_1 ; T_1^0 = (3;1; \ \frac{2}{3};0;1) ; \quad T_1 ; T_1^0 = (3;1;\frac{2}{3};0;1) ; \tag{3}$$

and

$$S_2 ; S_2^0 = (1;1;0;1;2) ; \quad S_2 ; S_2^0 = (1;1;0; \ 1;2) ;$$
$$T_2 ; T_2^0 = (1;3;0; \ \frac{2}{3};1) ; \quad T_2 ; T_2^0 = (1;3;0;\frac{2}{3};1) : \tag{4}$$

Compared to Ref. [4], we have introduced an extra set of messengers so as to mediate the SUSY breaking to both $SU(3)_1 \quad U(1)_{Y_1}$ and $SU(3)_2 \quad U(1)_{Y_2}$. Furthermore, there are three gauge-singlet superfields, X, Y and Z. Y is responsible for the SUSY breaking, X is related to the EWSB, and Z to the topcolor symmetry breaking.

The superpotential is written as follows,

$$\begin{aligned} W = {} & m_1 (S_1^0 S_1 + S_1^0 S_1) + m_2 (T_1^0 T_1 + T_1^0 T_1) + m_3 S_1 S_1 + m_4 T_1 T_1 \\ & m_1^0 (S_2^0 S_2 + S_2^0 S_2) + m_2^0 (T_2^0 T_2 + T_2^0 T_2) + m_3^0 S_2 S_2 + m_4^0 T_2 T_2 \\ & + Y ({}_1 S_1 S_1 + {}_2 T_1 T_1 + {}_1^0 S_2 S_2 + {}_2^0 T_2 T_2 \quad {}_1^2) \\ & + {}_3 X (H_u H_d \quad {}_2^2) + {}_4 Z [Tr({}_1 \ {}_2) \quad {}_3^2] ; \end{aligned} \tag{5}$$

where the Yukawa interactions are not written. It is required that $m_3^{(0)} = m_4^{(0)} \neq {}_1^{(0)} = {}_2^{(0)}$ so that the terms proportional to $m_3^{(0)}$ and $m_4^{(0)}$ cannot be eliminated by a shift in Y.

The model conserves the number of S_i-type and T_i-type ($i = 1;2$) fields. In addition, the superpotential has a discrete symmetry of $(S_i^{(0)};T_i^{(0)}) \leftrightarrow (S_i^{(0)};T_i^{(0)})$. The way of introducing the singlet fields X, Y and Z more naturally was discussed in Ref. [11] where these kind of fields are taken to be composite. Moreover, the Fayet-Iliopoulos D-terms for the $U(1)$ charges have been omitted. This is natural in the GMSB scenario. The above discrete symmetry and the exchange symmetry of ${}_1$ and ${}_2$ in the superpotential avoid such D-terms at the one-loop order.

The SUSY breaking is characterized by the term $\lambda_1^2 Y$ in Eq. (5). It is communicated to the observable sector through the gauge interactions by the messengers. The $SU(3)_2 \times U(1)_{Y_2} \times SU(2)_L$ are weak enough to be described in perturbation theory. Their gauginos acquire masses in the one-loop order [4,12],

$$
\begin{aligned}
M_{SU(3)_2} &= \frac{\alpha_3'}{4\pi}\Lambda_T\,,\\
M_{U(1)_{Y_2}} &= \frac{\alpha_1'}{4\pi}\left(\Lambda_S + \frac{2}{3}\Lambda_T\right),\\
M_{\tilde{W}} &= \frac{\alpha_2}{4\pi}\Lambda_S\,, \qquad (6)
\end{aligned}
$$

where $\alpha_i^{(\prime)} = g_i^{(\prime)2}/4\pi$ with g_3', g_1' and g_2 being the gauge coupling constants of the $SU(3)_2 \times U(1)_{Y_2} \times SU(2)_L$. For simplicity, we take $m_1^2 \simeq m_2^2 \simeq m_1'^2 \simeq m_2'^2 \simeq \lambda_{1,2}^{(\prime)}\Lambda_1^2 \simeq m_3^{(\prime)2} \simeq m_4^{(\prime)2}$ and $\lambda_1 \simeq \lambda_2 \simeq \lambda_1' \simeq \lambda_2' \simeq O(1)$. In this case, the Λ_S and Λ_T are approximately $\lambda_1\Lambda_1^2/m_1$. They are about 100 TeV by choosing, say $\Lambda_1 \sim 10^6$ GeV and $m_1 \sim 10\Lambda_1$. For the $SU(3)_1 \times U(1)_{Y_1}$, however, the gaugino masses cannot be calculated by the perturbation method, because the interactions are too strong. Nevertheless they should be at the order of $\lambda_1\Lambda_1^2/m_1$ given the above parameter choice,

$$
M_{SU(3)_1} \sim M_{U(1)_{Y_1}} \sim 100\ \text{TeV}\,. \qquad (7)
$$

Similarly, the first two generation scalar quarks and the electroweak Higgs particles obtain their masses in the two-loop order,

$$
\begin{aligned}
m^2_{\tilde{Q}_1} = m^2_{\tilde{Q}_2} &\simeq \frac{4}{3}\left(\frac{\alpha_3'}{4\pi}\right)^2\Lambda_T^2 + \frac{3}{4}\left(\frac{\alpha_2}{4\pi}\right)^2\Lambda_S^2 + \frac{1}{4}\left(\frac{\alpha_1'}{4\pi}\right)^2\left(\Lambda_S^2 + \frac{2}{3}\Lambda_T^2\right),\\
m^2_{\tilde{c}_R} = m^2_{\tilde{u}_R} &\simeq \frac{4}{3}\left(\frac{\alpha_3'}{4\pi}\right)^2\Lambda_T^2 + \frac{4}{9}\left(\frac{\alpha_1'}{4\pi}\right)^2\left(\Lambda_S^2 + \frac{2}{3}\Lambda_T^2\right),\\
m^2_{\tilde{s}_R} = m^2_{\tilde{d}_R} &\simeq \frac{4}{3}\left(\frac{\alpha_3'}{4\pi}\right)^2\Lambda_T^2 + \frac{1}{9}\left(\frac{\alpha_1'}{4\pi}\right)^2\left(\Lambda_S^2 + \frac{2}{3}\Lambda_T^2\right),\\
m^2_{h_u} = m^2_{h_d} &\simeq \frac{3}{4}\left(\frac{\alpha_2}{4\pi}\right)^2\Lambda_S^2 + \frac{1}{4}\left(\frac{\alpha_1'}{4\pi}\right)^2\left(\Lambda_S^2 + \frac{2}{3}\Lambda_T^2\right), \qquad (8)
\end{aligned}
$$

where Q_1 and Q_2 stand for the superfields of $(u, d)_L$ and $(c, s)_L$ respectively. And $(h_u\ h_d)$ are the scalar components of (H_u, H_d). Λ_S^2 and Λ_T^2 was calculated to be [4]

$$\Lambda_S^2 = \frac{4\lambda_1'^2\Lambda_1^4}{m_1'^2}\,; \qquad \Lambda_T^2 = \frac{4\lambda_2'^2\Lambda_1^4}{m_2'^2}\,: \tag{9}$$

For the third generation squarks and the topcolor Higgs' Φ_1 and Φ_2, the masses are around Λ_S^2 or Λ_T^2,

$$m^2_{\tilde{Q}_3} \sim m^2_{\tilde{t}_R} \sim m^2_{\tilde{b}_R} \sim m^2_{\Phi_1} = m^2_{\Phi_2} \sim \Lambda_S^2\,;\ \Lambda_T^2 \sim (100\ \mathrm{TeV})^2\,: \tag{10}$$

We have seen that for the super strong topcolor interactions, the relevant supersymmetric particles are super heavy ~ 100 TeV so that they decouple at the topcolor scale. The topcolor physics does not change even after the supersymmetric extension. However the topcolor Higgs fields seem to be too heavy.

Let us consider the breaking of the gauge symmetries. The $SU(3)_1 \times SU(3)_2 \times U(1)_{Y_1} \times U(1)_{Y_2}$ break into the diagonal subgroups $SU(3)_{QCD} \times U(1)_Y$ when the Higgs fields Φ_1 and Φ_2 get non-vanishing VEVs,

$$\langle\Phi_1\rangle = v_1\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \langle\Phi_2\rangle = v_2\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\,: \tag{11}$$

v_1 and v_2 are determined by the minimum of the following potential,

$$V_{topc} = |\lambda_4(3v_1v_2 - \Lambda_3^2)|^2 + \frac{g_1^2 + g_1'^2}{2}(v_1^2 - v_2^2)^2 + m^2_{\Phi_1}v_1^2 + m^2_{\Phi_2}v_2^2\,; \tag{12}$$

where g_1 is the coupling constant of the $U(1)_{Y_1}$. It is easy to see that in the case of $\lambda_4\Lambda_3^2 \gg m^2_{\Phi_i}$,

$$v_1 = v_2 = \frac{1}{\sqrt{3}}\left(\Lambda_3^2 - \frac{m^2_{\Phi_i}}{\lambda_4}\right)^{1=2}\,: \tag{13}$$

To keep v_1 and v_2 to be around a few TeV, certain fine-tuning for the scale Λ_3 is required in this model to cancel the 100 TeV m_{Φ_i}, where the coupling λ_4 is $O(1)$. The value of Λ_3 is more natural if the topcolor scale is higher, such as 10 TeV. However, it should be noted that raising topcolor scale makes the effective topcolor theory more tuned.

At the energy below the topcolor scale, the model is described by an effective theory in which the gauge symmetry groups are that of the SM, and there are two Higgs doublets and three generation quarks with four-fermion topcolor interaction for the third generation. In addition, there are weak scale gauginos of the SM, squarks of the first and second generations and doublet Higgsinos which become massive after the EWSB. There are also topcolor Higgsinos of $_1$ and $_2$ after the topcolor symmetry breaking. They typically have (1 10) TeV mass and are not expected to be important to the low energy physics. Because of the degeneracy between the first two generation squarks and the decoupling of the third generation squarks, this model is free from the SUSY FCNC problem.

The physics of the topcolor four-fermion interaction and the EWSB in this model is essentially the same as that without SUSY, which will not be discussed further.

IV. SUMMARY AND DISCUSSION

It has been known that the SUSY FCNC problem can be avoided if the squarks take the mass pattern that the first two generations with the same chirality are degenerate and the third generation is super heavy. We have constructed a supersymmetric topcolor model within GMSB to realize this mass pattern. The pattern is stable under the correction of the Yukawa interactions because they are weak and the third generation quarks obtain masses dynamically.

This model has therefore, the phenomenologies of both SUSY and topcolor. It predicts weak scale SUSY particles, like the SM gauginos, Higgsinos. It also predicts top pions. These predictions can be tested directly in the experiments in the near future. The indirect evidences of this model in low energy processes, such as in the B decays [13], and the R_b problem of it [14] are more complicated because of the involvement of both the SUSY and the topcolor, and deserve a separate study.

It should be addressed that this model has an inherent tuning problem. This required tuning follows from the very large masses (100 TeV) of the third generation scalars and

the topcolor Higgs. These fields are closely related to the topcolor and the EWSB scales which are, however, lower than 100 TeV. We have explicitly mentioned the tuning below Eq. (13). Another aspect of this tuning is that the naturalness of the EWSB requires the third generation scalars to be lighter than 20 TeV [6]. Note that the large mass 100 TeV is just a rough estimate due to that we are lack of nonperturbative calculation method. On the other hand, if we adjust the SUSY breaking scale and the messenger scale to be somewhat lower than what we have chosen, this problem can be less severe.

We emphasis that it is the degeneracy of the first two generations, rather than the heaviness of the third generation, that plays the essential role in solving the SUSY FCNC problem. In this sense, the consideration of this paper is less nontrival than the idea of effective SUSY. However, if we further consider the underlying theory, the models which realize effective SUSY [7,8] and the SUSY topcolor model of this paper are on an equal footing.

A comment should be made on the necessity of the supersymmetric topcolor. Although SUSY does not necessarily need the help from topcolor, their combination has certain advantages. As is well-known, SUSY keeps the weak scale, but cannot explain it. The weak scale may have a dynamical origin [15,16,11,17]. In this case, it is natural to expect that the physics which explains the fermion masses is also at some low energy. Topcolor provides such physics for the hierarchy between the third generation and the first two generations. On the other hand, SUSY maybe helpful to understand the hierarchy between the first and the second generation further. For instance, it is possible that the second generation quarks mainly get their masses from the electroweak Higgs VEVs, and the first generation quarks purely from the sneutrino VEVs [18].

Finally, it should be noted that the very heavy third generation squarks may pull up the light scalars. This pull up occurs through two- or more-loop diagrams with the topcolor Higgs exchanges. The heavy topcolor Higgs suppress this quantum correction. The suppression, however, may be not enough to keep the results of Eq. (8) from significant changing numerically. The fine-tuning problem which was discussed above re-appears here. In fact,

the drawback of the SUSY and topcolor combination is that the SUSY breaking scale and the topcolor scale are irrelevant. It might be hopeful to think of certain dynamics to make relation between them. For example, it is possible that the topcolor Higgs superfields are also the SUSY breaking messengers. This possibility will simplify the model and reduce the fine-tuning. It is reasonable to say that the supersymmetric topcolor is an interesting scenario which is worthy to be studied further.

ACKNOWLEDGMENTS

The author would like to thank many Korean colleagues for various helpful discussions.

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